

The logical choice for the related velocity defect law is

$$\frac{v_0}{u_\tau^2} (U_1 - U_0) = f_2 \left( \frac{x_2}{\delta} \right) \quad (4)$$

In this equation,  $U_0$  is the mainstream velocity. For turbulent asymptotic layers in zero pressure gradient ( $v_0 U_0 = -u_\tau^2$ ), Eq. (4) appears to agree with experimental data, as Fig. 2 shows.

It is seen that these asymptotic layers exhibit extended regions in which their mean velocity profiles are semilogarithmic. The slope of the logarithm can be represented by

$$w^* = x_2 \frac{\partial U_1}{\partial x_2} = -0.06 \frac{u_\tau^2}{v_0} \quad (5)$$

This relation, which probably holds for all turbulent boundary layers with moderate suction, does not conflict with the experimental data within the range  $0.04 < -v_0/u_\tau < 0.10$ , as is shown in Fig. 1.

The mean velocity profiles of Fig. 2 have practically no curved "tail" at the outer edge of the layer. Hence, Mickley and Smith's assertion that this tail (which is described by Coles' wake function) is unaffected by transpiration, cannot be extended to all turbulent boundary layers with suction. Finally, strict similarity of mean velocity profiles according to a velocity defect law can be expected only for "equilibrium layers" in the sense defined by Clauser.<sup>4</sup> The asymptotic layer is believed to be a proper equilibrium layer; the rather great spread in the data collected by Mickley and Smith at the larger blowing rate may partially be due to lack of equilibrium of the boundary layers concerned.

#### References

- <sup>1</sup> Mickley, H. S. and Smith, K. A., "Velocity defect law for a transpired turbulent boundary layer," AIAA J. 1, 1685 (1963).
- <sup>2</sup> Mickley, H. S. and Davis, R. S., "Momentum transfer over a flat plate with blowing," NACA TN 4017 (1957).
- <sup>3</sup> Rubesin, M. W., "An analytical estimation of the effect of transpiration cooling on the heat transfer and skin friction characteristic of a compressible turbulent boundary layer," NACA TN 3341 (1954).
- <sup>4</sup> Clauser, F. H., "The turbulent boundary layer," *Advances in Applied Mechanics* (Academic Press Inc., New York, 1956), Vol. 4, pp. 1-51.
- <sup>5</sup> Favre, A., Dumas, R., and Verollet, E., "Couche limite sur paroi plane poreuse avec aspiration," Publ. Sci. Tech. Min. Air 377 (1961).

## Reply by Authors to H. Tennekes

H. S. MICKLEY\* AND K. A. SMITH†

*Massachusetts Institute of Technology, Cambridge, Mass.*

THE authors would like to thank Tennekes for his interest in their earlier paper<sup>1</sup> and to extend their congratulations for his contribution to the technology of sucked boundary layers. However, they are concerned that Tennekes' interpretation of their paper may indicate a general misconception regarding it. A semilogarithmic mean velocity profile is not necessarily "characteristic of Clauser's phenomenological description." To the contrary, Clauser's deficiency law treatment<sup>2</sup> is valid only for the outer portion of the boundary layer and is, therefore, more nearly analogous

to Coles'<sup>3</sup> "Law of the Wake." For this reason, the authors refrained from drawing any conclusions with regard to the inner portion of the transpired boundary layer.

The authors concur with Tennekes' belief that their concept may not be applicable to sucked boundary layers. Their analysis assumed that the inner and outer portions of the boundary layers were coupled only loosely and that the maximum shear stress was representative of the stress applied to the inner face of the outer portion. For sucked boundary layers, the stress applied to the outer portion is not well defined, and the coupling assumption may be incorrect. For this reason, the original article was restricted to a discussion of transpired boundary layers.

#### References

- <sup>1</sup> Mickley, H. S. and Smith, K. A., "Velocity defect law for a transpired turbulent boundary layer," AIAA J. 1, 1685 (1963).
- <sup>2</sup> Clauser, F. H., "The turbulent boundary layer," *Advances in Applied Mechanics* (Academic Press Inc., New York, 1956), Vol. 4, pp. 1-51.
- <sup>3</sup> Coles, D., "The law of wake in the turbulent boundary layer," J. Fluid Mech. 1, 191-226 (1956).

## Comment on "Mathematical Analysis of Corotating Nose-Gear Shimmy Phenomenon"

WILLIAM J. MORELAND\*  
*Dayton, Ohio*

LAGRANGE'S equations offer advantages in almost all dynamics problems, but their versatility does not relieve the analyst of the burden of knowing precisely how the system functions; nor if, through design or oversight, inherent nonlinearities are omitted, can large-amplitude motions be computed even though the geometric nonlinearities (automatically contributed by the method) are retained.

In Ref. 1 it is suggested that the geometrically and kinematically nonlinearized equations of Lagrange be used to determine the wide-angle motion of a corotating nose gear. This is not possible. It is not possible because of a number of concurrent causes, any one of which, acting singly, could invalidate the equations. For example, with oscillation amplitudes beyond a few degrees, the lateral wheel forces become so large that the ground contact is caused to slip. Under this circumstance the equations employed in Ref. 1 no longer apply. Additional examples of a different nature are due to coulomb damping (particularly in the strut), hydraulic (nonviscous) damper action, and nonlinear elasticity in the tires and structure. None of these nonlinearities is introduced automatically by Lagrange's equations as are the geometric nonlinearities. Each must be deliberately incorporated at the beginning of the analysis. The sensitivity of amplitude in a neutrally stable vibration is marked. The most trifling disturbances bring about disproportionate changes in vibration amplitude. Hence the omission of such factors as degrees of freedom in the air frame and dynamic imbalance make amplitude computation meaningless.

Regardless of the analytic procedure adopted, the information gained is no more reliable than are the assumptions that describe the system. This is illustrated in the following examples.

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\* Professor, Chemical Engineering Department

† Assistant Professor, Chemical Engineering Department

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\* Consulting Engineer